Where Have Elementary Math Instruction and Materials Gone Wrong

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A Critique of Elementary Mathematics Instruction: The Case for Evidence-Based, Explicit Teaching and Practice

This paper examines the conflict between prevalent constructivist-based practices in elementary education and the robust findings of cognitive and behavioral science. It argues that the failure to utilize features such as explicit instruction and incorporate effective memory retention strategies—retrieval practice (massed, spaced, cumulative/mixed or interleaved)—is responsible for students' difficulties in developing mathematical skill fluency, including mastering standard algorithms, an important foundation of long-term mathematical success in problem solving. Furthermore, it critiques current elementary mathematics textbooks, citing specific examples inspired by one high-profile curricula, *Eureka Math* (Great Minds)[1], for overemphasizing inefficient strategies at the expense of necessary procedural mastery.

The Instructional Disconnect: Constructivism and Textbook Bias

Many current elementary curricula materials are heavily influenced by "**constructivism** and **reform mathematics**", which advocate for "**minimally guided instruction**" where students are expected to discover mathematical principles through complex problem-solving and inquiry. While fostering conceptual understanding is vital, both cognitive and behavioral science tell us this approach cannot work. Cognitive science tells us that unguided discovery places an overwhelming extraneous cognitive load on novice learners, contradicting the fundamental limitations of working memory (Kirschner et al., 2006). Behavioral psychology has demonstrated in experimental studies that errors during the acquisition stage of learning are deadly to discrimination (Touchette & Howard, 1984; Skinner, 2002), the type of learning upon which applied performance and understanding are built (Haring & Eaton, 1978).

A significant contributing factor to the lack of fluency (accuracy and speed) is the content and structure of modern math textbooks, which prioritize strategy proliferation over procedural efficiency. Cognitive science concludes the use of multiple strategies taught all together blocks the student's ability to build an efficient, working schema (Sweller, 1988). It is best to start with the standard algorithm. Behavioral psychology demonstrates that multiple strategies interferes with discrimination (Stokes & Osnes, 1989). The goals of fluency and computational efficiency are often delayed or sidelined by lengthy explorations of non-standard, multi-step methods.

In math education, the notion that conceptual understanding must precede procedural skill development ignores or misunderstands the science demonstrating that working on more conceptual-focused understanding benefits procedural skill development (e.g., teaching students to create equivalent quantities thereby making challenging problems easier to solve) and building

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[1] Eureka Math, developed by Great Minds, is a widely used program that emphasizes conceptual understanding, which is important, but provides less development of procedural knowledge, especially vital in the earliest grades

procedural skill development benefits conceptual understanding (creating equivalent fractions with common denominators solidifies the understanding of the identity property of multiplication, fractions as a form of division, and division as solving for an unknown factor along with many other understandings such as understanding base unit quantities of less than 1). The bidirectional relationship between doing and understanding in math is fairly well-established in the empirical literature (see Rittle-Johnson, 2017). Importantly, high levels of conceptual understanding are likely not achievable in the absence of computational fluency for a given skill.

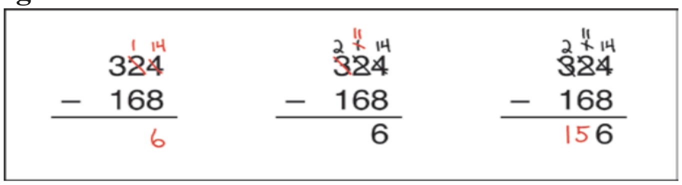
Examples from Elementary Curricula: Undermining Efficiency

The following examples, drawn from typical Grades 2 and 3 instructional materials, illustrate the focus on slow, multi-step strategies that consume cognitive resources instead of building automaticity with standard procedures:

Grade Level	**Inefficient Strategy (Curriculum Example)**	**Cognitive Critique (Why it's Problematic)**
Grade 2 Subtraction	**Arrow Way** (or *Counting On/Back*). To solve $570 - $110, students are taught to break the subtrahend into chunks and perform multiple jumps ($570 to 470 to 460).	This requires multiple steps and holds intermediate calculations in working memory, leading to high extraneous cognitive load. The standard vertical algorithm is the single most efficient method. The critical understanding is finding the missing addend, so subtraction needs to be taught as finding the difference.
Grade 2 Subtraction	**Compensation Strategy**. To solve $514 - $290, students change the problem to $524 - $300 by adding $10 to both numbers.	This requires an extra step of manipulating the numbers and requires conceptual reasoning that may be beyond students still developing basic math fact fluency and in turn distracts from the clear, reliable steps of the standard algorithm.
Grade 3 Multiplication	**Distributive Property with Area Models**. To solve $12 times $3, students must decompose $12 into $(10 times 3) + (2 times 3) and/or model this with an area diagram.	While this may be seen as key for conceptual grounding, using this lengthy method for basic facts delays the necessary fluency (automaticity) and consumes time that should be spent on fact memorization and the introduction of efficient multi-digit procedures.

These alternative methods, often treated as new algorithms, delay the introduction and mastery of the standard algorithms[2], undermining the fact that mathematics is relentlessly hierarchical. Students lacking fluency with efficient procedures will struggle to manage the computational demands of later coursework (Gersten et al., 2009). Figure 1 is an efficient example using the standard algorithm for subtraction.

Figure 1



The Math Imperative: Explicit Instruction, Durable Retention, Transfer and Generalization

Learning science mandates two things for successful learning: *how* information should be taught (explicit instruction) and *how* it must be reviewed for long-term retention (especially retrieval practice). The severe limitations of the brain's working memory require that initial instruction be **explicit**—clear, direct, and systematic.

- **Minimizing Cognitive Load:** By clearly modeling the expert procedure (schema) through explicit instruction, the teacher minimizes the extraneous cognitive load associated with unguided discovery. This ensures the student focuses their resources on encoding the correct information.
- **Reduced efficiency:** Kirschner et al. (2006) affirm that: "Minimally guided instruction is less effective and less efficient than instruction that guides students through the learning process."

Behavioral science explains the process as first acquisition where the student has to make the correct discrimination, and the teacher arranges the task and task presentation to promote correct responding with immediate error correction and faded support that is specifically designed to minimize the occurrence of errors. Only after the student has acquired the skill should the student be provided with fluency-building instruction which emphasizes a high dosage of opportunities to respond at the right level of task difficulty (Greenwood, 1991) which does a number of things—it increases the students' experience of reinforcement during instruction which improves their active engagement and motivation to respond, and it makes the response fluent which produces skill retention and endurance (Binder, 1996). Finally, such instruction increases the probability of applied performance and faster learning of more complex related skills (Burns, VanDerHeyden, & Jiban, 2006).

[2] Standard algorithms are sets of steps used to solve math problems, especially addition, subtraction, multiplication, or division of multi-digit numbers. Standard algorithms provide efficient methods for students to use throughout their grades. For example, multi-digit subtraction involves lining up numbers by place value, starting from the right (ones place), and subtracting each column, regrouping (borrowing) from the next column to the left if the top digit is smaller than the bottom digit, a process that breaks down a ten or hundred into ten ones or ten tens.

In addition to explicit initial instruction, careful and intentional practice is essential for long term retention. To achieve durable fluency, elementary math instruction must incorporate evidence-based practice and memorization strategies, including retrieval practice (massed for initial fluency, spaced for endurance, interleaved for transfer), task interspersal, incremental rehearsal, timed practice intervals with delayed corrective feedback and 'indiscriminable' contingencies[3].

Strategy	Description	Cognitive Rationale
Spaced Practice	Distributing practice of a concept or skill over increasing time intervals (e.g., reviewing Topic A after 3 days, 1 week, and 1 month).	**Prevents rapid forgetting** by forcing the memory system to work harder to recall the information after a gap, strengthening the memory trace (Dunlosky et al., 2013).
Interleaved Practice	Mixing different types of problems, concepts, or algorithms within a single practice session (e.g., solving division, then fractions, then perimeter).	**Develops discrimination** (schema selection); forces students to identify the correct strategy for a given problem, which is essential for problem-solving competence (Rohrer, 2015).
Retrieval Practice	Actively recalling information from memory, often via low-stakes quizzes or self-testing, without looking at notes.	**The act of retrieval itself is a powerful learning event** that reinforces the neural pathways, making the information easier to access in the future (Roediger & Karpicke, 2006).

Recommendations: A Call for Evidence-Based Reform

To align elementary mathematics education with the principles of the **Science of Math**, systemic reform is necessary across curriculum materials design and instructional practice.

1. **Instructional Mandate: Explicit instruction** must be the foundational teaching method for new concepts and procedures, with an immediate focus on teaching the most efficient standard algorithm. Teachers must consistently use modeling, guided practice, and immediate corrective feedback (Archer & Hughes, 2011).
2. **Curriculum Restructuring:** Textbook publishers must restructure content to reduce strategy proliferation and eliminate the unnecessary delay in teaching standard algorithms. Curricula must structurally integrate spaced, interleaved, and retrieval practice into daily and practice sessions that sample the content in a variety of "slices" from just acquired, to recently acquired, to review of previously mastered skills to ensure

[3] The purpose of indiscriminable contingencies is to promote generalization and maintenance of learning over time by making reinforcement unpredictable. This encourages the student to continue answering questions correctly even when a teacher is not present to provide immediate, consistent praise, because they have come to associate the *act* of doing math with the *potential* for future reinforcement (Freeland, J.T., Noell, G.H., 2002)

long-term retention. Instead of explicit instruction, many textbook publisher programs advocate for “productive failure” which causes students to develop math phobia.

3. **Prioritization of Fluency:** Educators and administrators must acknowledge that fluency with facts and efficient procedures is not an optional "old-school" skill, but a cognitive prerequisite that frees up working memory for complex problem-solving and conceptual reasoning in later grades. Rather than being at odds with creative, flexible, adaptable skill sets, fluency is the birthplace of applied performance (Johnson & Layng, 1992).

By shifting away from pedagogies that prioritize unguided exploration at the expense of cognitive efficiency, and by embedding memory science into instructional design, elementary education can ensure students acquire the robust and durable mathematical foundations necessary for academic success.

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